RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, AUGUST 2021
FIRST YEAR [BATCH 2020-23]

Date : 12/08/2021 **MATHEMATICS**
Time : 11am-1pm **Paper** : MACT 4 Full Marks : 50

Instructions to the students

- Write your **College Roll No, Year, Subject** & **Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject** & **Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group A

Linear Algebra 1

Unless mentioned all the symbols have their usual significance.

Answer all the questions, maximum one can score 30.

1. Find all solutions to the following system:

$$\begin{aligned} 2x+5y-6z-3w &= 7 \\ 2x-5y+6z-3w &= -3 \end{aligned}$$

[6]

2. Let $V=\{(x_1,x_2,x_3,x_4)\ \in \mathbb{R}^4 : x_1-5x_2+7x_3-4x_4=0\}$.

   (a) Show that $S=\{(-1,4,3,0)\}$ is a linearly independent subset of V. [1]

   (b) Extend S to a basis of V. [5]

3. Find the rank of the matrix: $\begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & -1 & 2 \\ 2 & 0 & 1 & 0 \\ 0 & -1 & 1 & -3 \end{pmatrix}$. [4]

4. Are the vectors $a_1 = (1, 1, 2, 4)$, $a_2 = (2, -1, -5, 2)$, $a_3 = (1, -1, -4, 0)$ and $a_4 = (2, 1, 1, 6)$ linearly independent in $\mathbb{R}^4$? Find a basis for the subspace of $\mathbb{R}^4$ spanned by a_1, a_2, a_3 and a_4.

[4+1]

5. (a) Prove that the only subspaces of $\mathbb{R}$ are $\mathbb{R}$ and the zero subspace. [2]

   (b) Prove that a subspace of $\mathbb{R}^2$ is $\mathbb{R}^2$ or the zero subspace or consists of all scalar multiples of some fixed vector in $\mathbb{R}^2$. [5]

6. Let V be the vector space of all $n \times n$ matrices over the field F, and let B be a fixed $n \times n$ matrix. If $T(A) = AB - BA$; check whether T is a linear transformation from V into V. [3]

7. Let V be a n-dimensional vector space over the field F and let T be a linear transformation from V into V such that the range and null-space of T are identical. Prove that n is even. [2]

8. Find two linear operators T and U on $\mathbb{R}^2$ such that $TU = 0$ but $UT \neq 0$. [3]

Group B

Applications of Differential Calculus

Answer any four questions from question no. 9−14. [4 x 5]

9. Show that the point of intersection of the curve $2y^3 - 2x^2y - 4xy^2 + 4x^3 - 14xy + 6y^2 + 4x^2 + 6y + 1 = 0$ and its asymptotes lie on the line $8x + 2y + 1 = 0$.

10. Find the envelope of the family of circles whose centres lie on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and which pass through its centre.

11. Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is an another cycloid.

12. Find the intervals in which the curve $y = e^x(\cos x + \sin x)$ is concave upwards or downwards, $x \in (0, 2\pi)$.

13. Find the pedal equation of the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ with respect to origin.

14. Find the position and nature of the double points of the curve $y(y-6) = x^2(x-2)^3-9$. Find also the equation of the tangent at the double point if the tangent is real.

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